# Nuclear Physics - A Summary

The characteristic scale for atomic radius is an ngstrom $10^{-10}$m. For nuclear radius, it is a fermi $10^{-15}$m.

We have the relation, from experiments

$$R = r_0 A^{1/3} \qquad r_0 \equiv 1.2fm \tag{0.1}$$

## 1 Cross-Sections

If a sphere, radius $R_1$, is incident upon another, radius $R_2$, and the distance between the centres $b$ (the *impact parameter*). Then, what happens depends on $b$:

- $b > R_1 + R_2$: nothing;
- $b < R_1 + R_2$: some interaction.

The *effective cross section* (of a single target atom) is given by:

$$\sigma_T = \pi(R_1 + R_2)^2 \tag{1.1}$$

If a beam of $N_0$ atoms is incident upon a target, width $x$, area $A$; having number density of atoms within target $n$. Let $\Delta N$ be the number of beam atoms that interact.
The *total effective cross section* is:

$$\sigma_T n x A$$

The *probability of interaction* is:

$$\frac{\sigma_T n x A}{A} = \sigma_T n x$$

Hence, the number of interacting beam particles is $\Delta N = N_0 \sigma_T n x$. That is, we have found the *reaction rate*.

## 2 Nuclear Charge Distribution

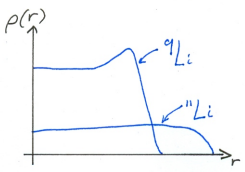


Figure 1: Charge distribution

Parameterise the nuclear radius:

- Where $\rho(r) \to \rho(0)/2$;
- *Skin thickness* $t$: where the density goes from 90% of central, to 10%;
- Mean square radius:

$$\langle r^2 \rangle = \frac{\int r^2 \rho(r) d\tau}{\int \rho(r) d\tau} \tag{2.1}$$

Special case: lithium-11: it has a *halo nucleus*; the extra couple of neutrons (w.r.t Li-9) form a neutron-rich skin, extending the radius to something comparable with lead.

# 3 Differential Cross-Sections

Beam, incident upon an effective area $d\sigma$, within target, scatters into a cone, $d\Omega$, at angle $\theta$ to beam direction. This gives a *differential scattering cross-section*:

$$\frac{d\sigma}{d\Omega}(\theta)$$

Then, the total cross-section is just the integral:

$$\sigma = \int d\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \tag{3.1}$$

Where $d\Omega = \sin\theta d\theta d\phi$. The pattern seen, on a plot of $\frac{d\sigma}{d\Omega}$ against $\theta$ is akin to Fraunhoffer diffraction. The pattern gives information on the scattering surface.
For an opaque disc, diameter $D$, the first minimum will be at $1.22\frac{\lambda}{D}$. The nuclear surface is neither well defined or opaque, hence, first minimum does not go to zero.

$$\theta = 1.22\frac{\lambda}{D} \tag{3.2}$$

If we consider the overlap integral, for some incoming wave $\psi_i$ and outgoing $\psi_f$, incident upon a potential:

$$M_{if} = \int \psi_f^* V(r) \psi_i d\tau$$

Then, we have that:

$$\frac{d\sigma}{d\Omega} \propto M_{if}^2 \tag{3.3}$$

If we let the *momentum transfer vector*, for elastic scattering ($|\boldsymbol{k}_i| = |\boldsymbol{k}_f| = k$) be $\boldsymbol{q}$. Then:

$$\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f \quad \Rightarrow \quad q = 2k \sin\frac{\theta}{2} \tag{3.4}$$

Then, we can derive that:

$$M_{if} = \int V(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}} d\tau$$

That is, if the scattering happens due to the nuclear charge $\rho(\boldsymbol{r})$. then we have that $M_{if} = \mathcal{F}(\rho)$; the 3D Fourier transform. Hence:

$$\frac{d\sigma}{d\Omega} \propto \mathcal{F}^2\{\rho(\boldsymbol{r})\} \tag{3.5}$$

The FT is usually defined as the *electronic form factror of the nucleus.* That is: $F(q) \equiv \mathcal{F}\{\rho(\boldsymbol{r})\}$.

If we use an extended charge distribution, then the cross section is related to the FT of the charge distribution, and the CS due to a point-charge nucleus via:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{point} F^2(q) \tag{3.6}$$

Then, inversion of the FT (i.e. experimental data) will give us the potential that the beam scattered from. That is, we can find the nuclear charge distribution $\rho(\boldsymbol{r})$. There will be errors on $\rho$, at small $r$, as $q$ will be needed to be known up to infinity to get it exactly right. Which is impossible.

## 3.1 Conclusions

$\rho(r = 0)$ is similar for all nuclei;

The nuclear surface is diffuse.

## 3.2 $\rho(r)$ From Perturbations

Look at the energy differences in electron transitions, for real & point nucleus. There will be the greatest effect for the electron with w.f. with most penetration of the nucleus. This does not work, as not point nuclei! The energy shift is very small, and we cannot compute w.f's to be able to compare.

So, use X-rays isotope shifts:

### 3.2.1 X-ray Isotope Shifts

Measure the energy of an electron K X-ray, in two different isotopes. Use K X-ray, as most overlap with nucleus.

Look at differences in energies.
This subtracts out the point nucleus, and has a larger (but still very small) energy shift:

$$\Delta E \approx 100keV \qquad \delta E \approx 0.15eV \tag{3.7}$$

Where $\Delta E$ is the transition energy, and $\delta E$ the shifts.

The shifts will be due to differing $\rho(r)$.

### 3.2.2 Optical Isotope Shifts

Measure the difference in energies in optical transitions. Here:

$$\Delta E \approx 3eV \qquad \delta E \approx 10^{-6}eV \tag{3.8}$$

The shift is smaller, as optical transition levels will have less overlap with nucleus.

This shift is able to be measured to great precision: use photomultiplier tubes, having almost single atom sensitivity. Tune a laser to excite atoms, and see spectrum.
This is the only method that can be used on short lifetime nuclei (such as Li-11).

All shifts discussed thus far are small, as nuclear radius & Bohr radius of completely different scales. So, use Muonic atoms:

### 3.2.3 Muonic Atoms

Replace electrons in an atom, with muons.

There is a huge $\delta E$, because $\mu$ w.f. peaks inside the nucleus; hence, most affected by nuclear charge distribution. That is:

$$|\psi^{\mu}_{1s}(r=0)|^2 >> |\psi^{e}_{1s}(r=0)|^2 \tag{3.9}$$

# 4 Nuclear Matter Distribution

We use a probe that interacts via the strong nuclear force: $p, \pi, \alpha$.

If the probe energy is below the Coulomb repulsion, then 'Rutherford Scattering'. If energy above Coulomb repulsion, then deviation from RS.

The analysis of such data is much more complex than that for electron-scattering; because the strong force is very complicated.

Again, we see Fraunhoffer diffraction patterns, which gives information about the scattering surface; and hence the internal density distribution.

If we fire pions at a nucleus, they will spiral inwards, emitting $\pi$-mesic X-rays. At the nuclear surface, the pion will be absorbed. Hence, a way to find $R$.

### 4.0.4 Conclusions

The charge & matter distributions are very similar;

If there is a large $n$ excess, then halo nucleus formed;

Central density for all stable nuclei, is similar;

All have similarly diffusive surface: the skin thickness is similar for all stable nuclei.

# 5 Nuclear Models

## 5.1 Liquid Drop Model

This is a classical model.

Uses the SEMF.

The *binding energy* is the energy needed to break an atom into its constituents.

The mass of a nucleus is the mass of its constituents minus its BE.

$$BE = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta \tag{5.1}$$

Important features of the BE/A plot:
Most have about 8MeV per nucleon. The maximum BE is at iron (A = 56). He lies significantly away from it: being very bound.

Energy from fusion/fisson:

$$Q = \sum m_{initial} - \sum m_{final} \tag{5.2}$$

### 5.1.1 Deviations from Liquid Drop Model

There is extra BE for $Z, N$ =28, 50, 82, 126;

The $p, n$ separation energies highest for $8, 20, 28, 50, 82, 126$;

Nuclear excitation energies highest for above numbers.

*Magic Numbers*:

$$8, 20, 28, 50, 82, 126 \tag{5.3}$$

## 5.2 Shell Model

Protons & neutrons fill shells & orbitals.

*Woods-Saxon* potential:

$$V(r) = -\frac{V_0}{1 + e^{(r-R)/a}} \tag{5.4}$$

Where $a$ = 0.6fm, the surface diffusivity parameter; and $R$ the mean-square-radius. $V_0$ is about 35MeV.

We must then solve the TISE with this as the potential.

If we initially approximate using the Ho ($\frac{1}{2}m\omega^2 r^2$), then get ok shells, till about 20. The HO shells are less bound than reality.

We must also incorporate *spin-orbit* interaction.

A state $j$ is split into $j' = \ell - s$ and $j = \ell + s$. With $\epsilon_{j'} > \epsilon_j$.

$$\epsilon_j = -V_{SO}\langle \ell \cdot s \rangle \tag{5.5}$$

We do the standard computation of:

$$\langle \ell \cdot s \rangle = \frac{\hbar^2}{2}[j(j+1) - \ell(\ell+1) - s(s+1)] \tag{5.6}$$

The total energy shift: $\Delta E = \epsilon_{j'} - \epsilon_j$. This is (with $s = 1/2$):

$$\Delta E = \frac{V_{SO}\hbar^2}{2}(2\ell + 1) \tag{5.7}$$

This effect will mix levels; enough to produce closed shells at the magic numbers.

That is, the SO coupling, with WS potential, produces significant energy gaps, for occupation numbers at the magic numbers.