Diffraction:

Source at S, with a wavefront at Q, being a distance s away. There is an observation plane at P, distance r.

Thus, the electric field at Q is like:

$$E(Q) \propto \frac{e^{iks}}{s}$$

And, the contribution to the field at P, from the wavelet at Q is:

$$dE = k(a)e' \frac{e^{iks}}{s} \frac{e^{ikr}}{r} dA$$

Where $k(a)$ is some obliquity factor; e' the field strength per unit area, and is assumed to be constant. Hence, the *Huygen's-Fresnel Integral* is:

$$E(P) = \frac{e' e^{iks}}{s} \iint k(a) \frac{e^{ikr}}{r} dA$$

Where dA is the portion of unobstructed area of the aperture.

Fraunhofer Diffraction:

This is an example of "far-field" diffraction: that is, P is a long way from S. The wavefront is assumed to be a plane wave at the aperture. The following simplifications are made to the *HFI*; with R being the distance from the aperture to the observation plane.

$k(a) = 1$

$r \sim R$ (on denominator only)

Thus, absorbing all constants into the e factor:

$$E(X,Y) = \frac{e}{R} \iint e^{ikr} dA$$

Infact, the electric field at the observation plane is the Fourier transform of the aperture function (where X_i denotes observation plane coordinate and x_i on the aperture plane):

$$E(X,Y) = F\{A(x,y)\}$$

But being careful about constants. Now, for Cartesian, and useful for rectangular apertures:

$$E(X,Y)=\frac{ee^{ikR}}{R}\iint_{-\infty\to+\infty}A(x,y)e^{-\frac{ik}{R}(xX+yY)}dxdy$$

The intensity pattern for this, with an aperture function:

$$A(x)=\begin{cases}1 & |x|\leq a/2\\ 0 & |x|>a/2\end{cases}$$

Yields an intensity function:

$$I(X)\propto \mathrm{sinc}^{2}(X)$$

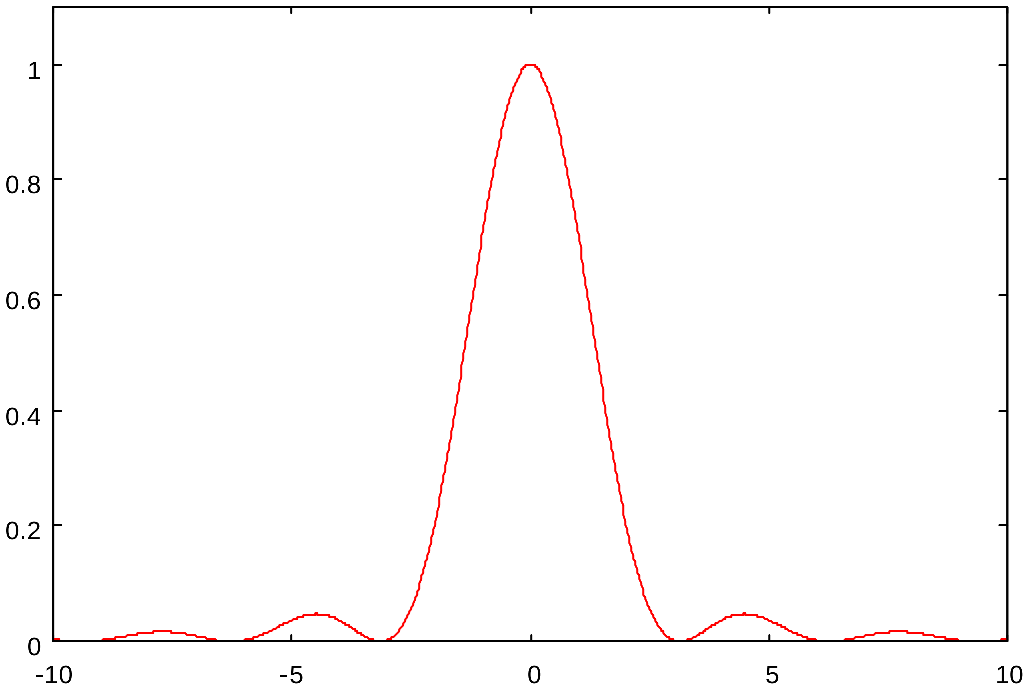

Figure 1. Intensity pattern for single thick slit, width a.

Or, for a circular aperture, using polar coordinates, we get a Bessel function:

$$E(U)=\frac{ee^{ikR}}{R}2p\int_{0}^{\infty}A(u)J_{0}\left(\frac{kuU}{R}\right)udu$$

This will produce an intensity pattern like:

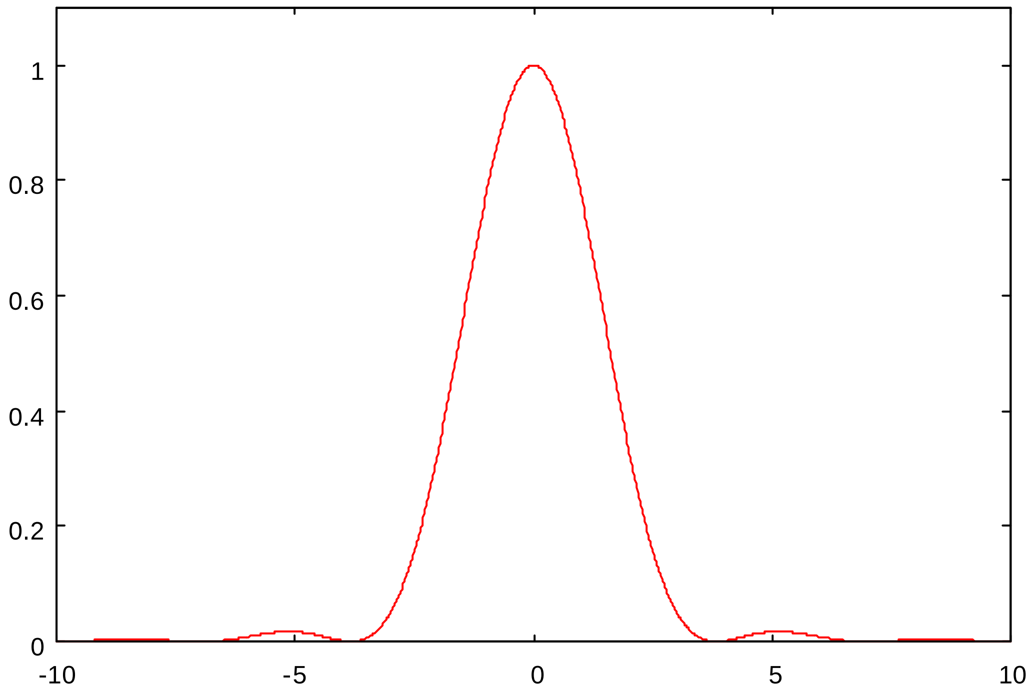

Figure 2. Intensity pattern for circular aperture

This was found from running through the algebra to get:

$$I(U) \propto I(0)\left[\frac{2J_1\left(\frac{kdU}{R}\right)}{\frac{kdU}{R}}\right]^2$$

Notice how most of the energy is concentrated in the centre. This central "disc" is known as the "Airy Disc". The first minimum of the function defines the angular size of the Airy disc, if the aperture has diameter *d*, and focal length *f*; with radius:

$$q_D = 1.22\frac{l}{d} \qquad\qquad R_A = fq_D$$

The above value for q_D gives the ultimate optical resolution of an optical instrument with aperture diameter *d*, observing light with a wavelength l .

So far, Fraunhofer diffraction has been for a single thick slit, and it has been seen that the electric field is the Fourier transform of the aperture function $E = F\{A\}$.

Suppose we consider the aperture function for two thin slits, separated by a distance d:

$$A_1(x) = d\left(x - \tfrac{d}{2}\right) + d\left(x + \tfrac{d}{2}\right)$$

Where $$d(x - x_0) = \begin{cases} 1 & x = x_0 \\ 0 & x \neq x_0 \end{cases}$$

This yields an intensity pattern like $\cos^2(xd)$.
Also, consider the aperture function for a single thick slit, of width a:

$$A_2(x) = \begin{cases} 1 & |x| \leq a/2 \\ 0 & |x| > a/2 \end{cases}$$

Now, the total aperture function for two thick slits, of width a, at a distance d apart can be given by the convolution:

$$A(x) = A_1(x) * A_2(x) = \int_{-\infty}^{\infty} A_1(x') A_2(x - x') dx$$

Hence, the electric field is given by the product of their transforms:

$$E = F\{A(x)\} = F\{A_1(x) * A_2(x)\} = F\{A_1(x)\} \cdot F\{A_2(x)\}$$

Thus, the total intensity field due to two thick slits can be written:

$$I(X) = I_0 \cos^2\left(\frac{kdX}{2R}\right) \text{sinc}^2\left(\frac{kaX}{2R}\right)$$

Which looks something like, for the case of $d = 10a$:

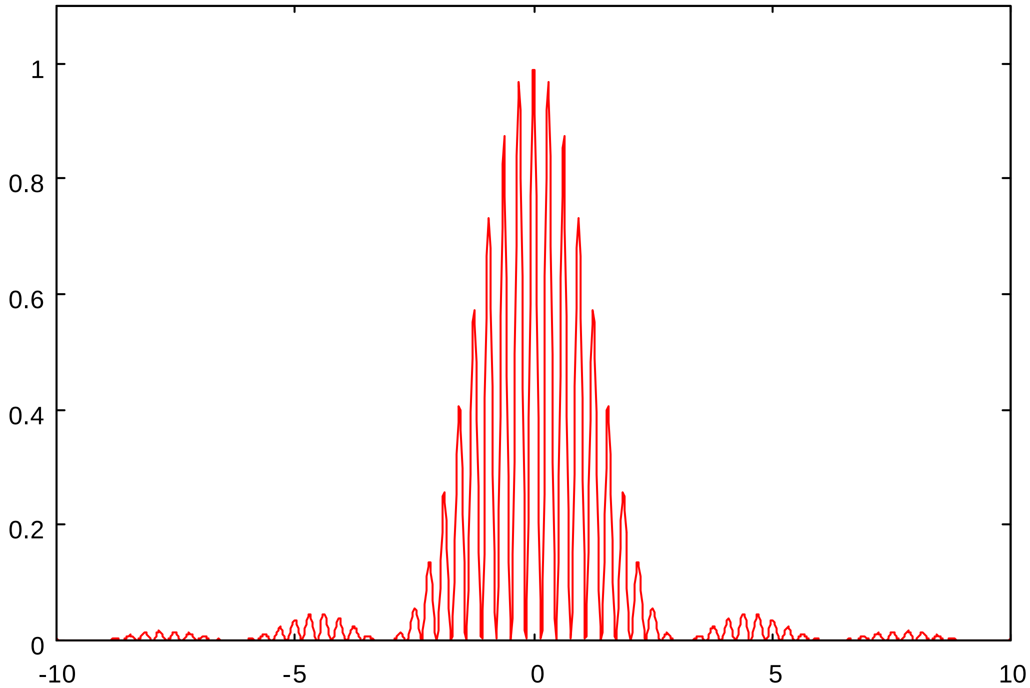

Figure 3. Intensity pattern for two thick slits; where the slit separation is 10x the slit width