Interference:

Consider:

$$\underline{E} = \underline{E}_1 e^{i(\underline{k}_1 \cdot \underline{r} - w_1 t + f_1)} + \underline{E}_2 e^{i(\underline{k}_2 \underline{r} - w_1 t + f_2)}$$

Which is a superposition of two plane waves.
Now, the intensity is given by the time average, over some period *T*

$$I = \langle EE^* \rangle_T$$
$$= E_1^2 + E_2^2 + 2\underline{E}_1 \cdot \underline{E}_2 \underbrace{\cos[(\underline{k}_1 - \underline{k}_2) \cdot \underline{r} - (w_1 - w_2)t + (f_1 - f_2)]}_{\equiv f(\underline{r}, t)}$$

Now, the "interference term" is $f(\underline{r}, t)$. Notice that if:

$\underline{E}_1 \cdot \underline{E}_2 = 0$ then no interference patter will be observed.

Thus, the first condition for interference is that:

$$\boxed{\underline{E}_1 \cdot \underline{E}_2 \neq 0}$$

We can write the interference term as:

$$f(\underline{r}, t) = \cos(\Delta \underline{k} \cdot \underline{r} + \Delta f)\cos(\Delta wt) + \sin(\Delta \underline{k} \cdot \underline{r} + \Delta f)\sin(\Delta wt)$$

Where: $\Delta \underline{k} \equiv \underline{k}_1 - \underline{k}_2 \quad \Delta f \equiv f_1 - f_2 \quad \Delta w \equiv w_1 - w_2$

Now, going back to the intensity expression, with the interference term written in:

$$I = E_1^2 + E_2^2 + 2\underline{E}_1 \cdot \underline{E}_2 \langle f(\underline{r}, t) \rangle_T$$

And again, the time average of the interference term is only non-zero (due to the sine & cosine terms) if:

$$w_1 \approx w_2 \Rightarrow \boxed{\Delta w \approx 0}$$

Which is the second condition for interference.
Thus, the interference term becomes:

$$f(\underline{r}, t) \rightarrow \cos(\Delta \underline{k} \cdot \underline{r} + \Delta f)$$

Again, which has non-zero time average when Δf is constant in time, over the averaging period *T*. Thus, the third condition for interference:

$$\boxed{\Delta f \neq f(t)}$$

Coherence $\equiv$ waves that maintain a fixed phase relationship

Temporal coherence…

If $\Delta w \neq 0$, then phase relationship will change over period $\sim 1/\Delta w$

e.g. if bandwidth $\Delta n = \dfrac{\Delta w}{2p}$, then:

coherence time: $t_c = \dfrac{1}{\Delta n}$

coherence length: $\ell_c = c t_c$

Coherence length is the distance over which beams will stay coherent… it is the greatest path length difference for beams to still produce an interference pattern.

Spatial coherence…

… maintain a fixed phase relationship over a wavefront

… important for "wavefront division"

Summarising the conditions for interference:

$$\underline{E}_1 \cdot \underline{E}_2 \neq 0$$
$$\Delta w << T^{-1}$$
$$\Delta f \neq f(t)$$