

Waveforms:

Plane waves:
$$E(\underline{r}, t) = \underline{E}_0 e^{i(\underline{k} \cdot \underline{r} \pm \omega t)}$$

Wavefront: surface of constant phase... i.e. $\underline{f} = \underline{k} \cdot \underline{r} \pm \omega t = \text{const}$

Spherical Waves:
$$E = \frac{A}{r} e^{\pm i(kr \pm \omega t)}$$

$$v_p = \frac{\omega}{k} \quad v_g = \frac{d\omega}{dk} \quad \text{“phase first”}$$

Spectra:

Now, as $E = E_0 e^{-i\omega t}$ is a solution, so is a linear superposition. Thus:

$$E = \sum_n A_n e^{-i\omega_n t}$$

Thus, can take this up to infinitesimals, and get the inverse Fourier Transform:

$$E(t) = \int_{-\infty}^{\infty} A_{\omega}(\omega) e^{-i\omega t} d\omega \quad \text{Inverse FT}$$

$$A_{\omega}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt \quad \text{FT}$$

Hence the spectrum of light is the Fourier Transform of the electric field. Similarly, for spatial:

$$E(x) = \int_{-\infty}^{\infty} A_k(k) e^{ikx} dk \quad \text{Inverse FT}$$

$$A_k(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(x) e^{-ikx} dx \quad \text{FT}$$

Huygen's Wavelets:

Every point on a wavefront acts as a point source of secondary spherical waves; these waves interfere to produce the next wavefront.

Fermat's Principle:

Light takes minimum time to travel between two points.

i.e. it minimises L :
$$L = \sum_i n_i \ell_i$$

ℓ_i is the path length of a ray, in a material of refractive index n_i .

Polarisation States:

Suppose we have the ray:

$$\underline{E} = \underline{\hat{x}}E_x e^{i(kz - \omega t)} + \underline{\hat{y}}E_y e^{i(kz - \omega t + \phi)}$$

Then:

Linear polarisation:

$$\begin{array}{ccc} \phi = 0 & \text{or} & \phi = \pi \\ \text{Inphase} & & \text{antiphase} \end{array}$$

Circular polarisation:

$$E_x = E_y \quad \phi = \frac{\pi}{2}$$

$$\phi = +\frac{\pi}{2} \quad \text{LH}$$

$$\phi = -\frac{\pi}{2} \quad \text{RH}$$

Elliptical polarisation:

ϕ arbitrary

Polarisation by reflection/scattering:

- from dipole emission... hence nothing emitted // to original E-field

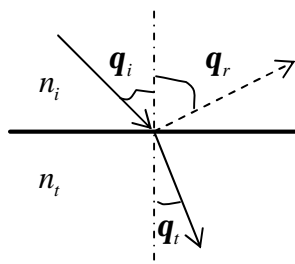
Reflectivity is polarisation dependant:

If \underline{E} is polarised // to plane of incidence, then p-polarised.

If \underline{E} is polarised perpendicularly to plane of incidence, then s-polarised.

Brewster Angle:

At certain angles of incidence, the angle of induced dipole will be // to direction of incidence.... Thus nothing reflected.



Zero amplitude reflected when:

$$q_r + q_t = 90^\circ$$

Thus: $q_i + q_t = 90$

Now, $q_i \equiv q_B$ "Brewster Angle"

Hence: $q_B + q_t = 90 \Rightarrow q_t = 90 - q_B$

Snell's law gives:

$$\begin{aligned}
 n_i \sin \mathbf{q}_i &= n_t \sin \mathbf{q}_t \\
 n_i \sin \mathbf{q}_B &= n_t \sin \mathbf{q}_t \\
 &= n_t \sin(90 - \mathbf{q}_B) \\
 &= n_t \cos \mathbf{q}_B
 \end{aligned}$$

Thus: $\frac{\sin \mathbf{q}_B}{\cos \mathbf{q}_B} = \frac{n_t}{n_i}$

Hence, the Brewster Angle is given by:

$$\mathbf{q}_B = \tan^{-1} \left(\frac{n_t}{n_i} \right)$$

Dichroism:

Some materials selectively absorb one polarisation state

E // to “long axis” does work in moving charge & is absorbed

E perp to “long axis” passes through

“long axis” is the axis of the polaroid... how molecules are arranged

Thus, transmitted field component:

$$E \cos \mathbf{q}$$

Transmitted intensity:

$$I(\mathbf{q}) = I_0 \cos^2(\mathbf{q})$$

Malus' Law

Optical Anisotropy:

- materials with different permittivities in different directions

$$\mathbf{e}_x \neq \mathbf{e}_y \neq \mathbf{e}_z$$

Hence, $\underline{\underline{\mathbf{e}}}$ will be some matrix quantity

If: $\mathbf{e}_x = \mathbf{e}_y \neq \mathbf{e}_z$ UNIaxial crystal

If: $\mathbf{e}_x \neq \mathbf{e}_y \neq \mathbf{e}_z$ BIaxial crystal

Recall: $\underline{D} = \underline{\underline{\mathbf{e}}}\underline{E}$, so now this is:

$$\underline{D} = \underline{\underline{\mathbf{e}}}\underline{E}$$

Hence, $\nabla \cdot \underline{E} \neq 0$, thus have to rederive the wave equation:

$$\nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E} = \underline{\underline{m}}_0 \underline{\underline{e}} \ddot{\underline{E}}$$

For a plane wave $\underline{E} = \underline{E}_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}$, can show that this becomes:

$$-k(\underline{k} \cdot \underline{E}) + k^2 \underline{E} = \omega^2 \underline{\underline{m}}_0 \underline{\underline{e}} \underline{E}$$

Now, the refractive index n_i of light which corresponds to travel in the direction in which the permittivity is $\underline{\underline{e}}_i$, is linked by $n_i^2 = \underline{\underline{e}}_i$. Thus, for the case of a uniaxial crystal:

$$\underline{\underline{e}}_x = \underline{\underline{e}}_y = n_o^2 \quad \underline{\underline{e}}_z = n_e^2$$

Thus, two solutions to the wave equations, each with different refractive indices.

Birefringence: an anisotropic material with two refractive indices.

For the solution with refractive index n_o : ordinary ray “o-ray”
 For the solution with refractive index n_e : extraordinary ray “e-ray”

The refractive index of the e-ray depends on \underline{q} .
 However, when $\underline{q} = 0$, $n_o = n_e \dots$ which is defined as the “*optic axis*”.... When e-ray & o-ray become indistinguishable.

Note again: if $\underline{\underline{e}}_x = \underline{\underline{e}}_y \neq \underline{\underline{e}}_z$ one optic axis uniaxial
 if $\underline{\underline{e}}_x \neq \underline{\underline{e}}_y \neq \underline{\underline{e}}_z$ two optic axes biaxial

e- & o-rays have the following properties:

$\underline{k} \cdot \underline{E}_o = 0$	o-ray perp to \underline{k}
$E_{o_z} = 0$	o-ray perp to optic axis
$\underline{E}_o \cdot \underline{E}_e = 0$	o-ray & e-ray are perpendicular to each other
$\underline{k} \cdot \underline{E}_e \neq 0$	(unless $\underline{q} = 0$, then $\underline{E}_o = \underline{E}_e$)

Now, $\underline{S} = \underline{E} \times \underline{B}$ (ish... with a factor of the permeability...)
 Poynting

If: $\underline{E} = \underline{E}_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}$ $\underline{B} = \underline{B}_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}$

Then: $\nabla \times \underline{E} = \underline{k} \times \underline{E}$ $\dot{\underline{B}} = -i\omega \underline{B}$ $\underline{k} \times \underline{E} = -i\omega \underline{B}$
 from the Maxwell relation $\nabla \times \underline{E} = -\dot{\underline{B}}$

Hence, can rewrite the Poynting vector:

$$\underline{S} = \frac{1}{i\omega} (\underline{E} \times \underline{k} \times \underline{E}) = \frac{1}{i\omega} (\underline{k}(\underline{E} \cdot \underline{E}) - \underline{E}(\underline{E} \cdot \underline{k}))$$

Thus, notice that for the o-ray, $\underline{E} \cdot \underline{k} = 0 \quad \therefore \quad \underline{S} // \underline{k}$

Notice also that for the e-ray, $\underline{E} \cdot \underline{k} \neq 0 \quad \therefore \quad \underline{S}$ not (generally) $//$ to \underline{k}
(only for $\mathbf{q} = 0$ again)

Now, when an \underline{E} -ray is incident upon a uniaxial crystal (not up optic axis), we get 2 beams... o-ray & e-ray... going in different directions... “double refraction”

Example of uniaxial crystal: calcite

Polarising beam splitter:

Suppose we stick two pieces of calcite together, which we can arrange so that the o-ray will undergo total internal reflection at the boundary of the two pieces, and that the e-ray passes through.

Remember that for TIR, the critical angle is dependant upon the refractive index, so the critical angle will thus be different for the o- & e-rays. $\mathbf{q}_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$.

Hence, only one polarisation state leaves the “beam splitter”.

Waveplates:

“fast”... $E_{//}$ lags E_{\perp} by \mathbf{d} $n_{\perp} > n_{//}$

“slow”... E_{\perp} lags $E_{//}$ by \mathbf{d} $n_{//} > n_{\perp}$

Halfwaveplate... $\mathbf{d} = \mathbf{p}$

Quarterwaveplate... $\mathbf{d} = \mathbf{p}/2$

- used to flip polarisation direction about optic axis.

Faraday Effect:

Plane of polarisation is rotated as light passes through a material in the presence of a \underline{B} -field. Rotation angle is given by:

$$\mathbf{b} = VBd \quad V = \text{“Verdet’s constant”} \quad B = \text{mag.field strength}$$

$$d = \text{path length through material}$$

rotation is independent of direction, so, if reflected back, then $\mathbf{b}' = 2\mathbf{b}$