Maxwell's relations:

Fundamental thermodynamics relation:

$$dE = TdS - pdV$$

Hence: $E = E(S,V)$

Thus: $$dE = \left(\frac{\partial E}{\partial S}\right)_V dS + \left(\frac{\partial E}{\partial V}\right)_S dV$$

Which shows that:

$$\left(\frac{\partial E}{\partial S}\right)_V = T \qquad \left(\frac{\partial E}{\partial V}\right)_S = -p$$

However, by the partial derivative rules:

$$\left(\frac{\partial}{\partial V}\right)_S \left(\frac{\partial E}{\partial S}\right)_V = \left(\frac{\partial}{\partial S}\right)_V \left(\frac{\partial E}{\partial V}\right)_S$$

Giving:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$$

Maxwell's 1st relation

From enthalpy: $dH = TdS + Vdp$

Similarly gives:

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$$

Maxwell's 2nd relation

From Helmholtz: $dF = -SdT - pdV$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

Maxwell's 3rd relation

From Gibbs: $dG = -SdT + Vdp$

$$\left(\frac{\partial S}{\partial p}\right)_p = -\left(\frac{\partial V}{\partial T}\right)_p$$

Maxwell's 4th relation