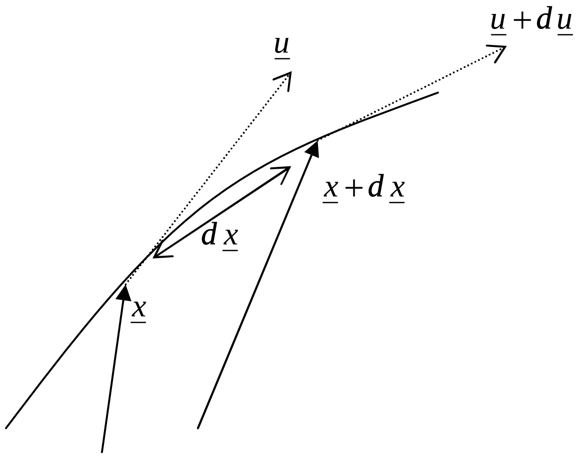
Concentrate on: inviscid;
Incompressible.

$$\underline{x} = (x, y, z) \qquad \underline{u} = (u, v, w) \qquad \underline{r} = (\underline{x}, t)$$

Lagrangian description: tracks particular particle.
Eulerian description: look at a "window" in space.

Stream Lines:



As $d\underline{x} \to 0$, $d\underline{x}$ & $\underline{u}$ become more and more parallel.
Thus, the streamline will have $\underline{u}$ as a tangent vector.
So:

$$d\underline{x} \propto \underline{u}$$

Hence, in the limit $d\underline{x} \to 0$:

$$d\underline{x} = k\underline{u}$$

That is:

$$(dx, dy, dz) = k(u, v, w)$$

Hence:

$$k = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Therefore, the streamline equation is given by:

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

When $u = v = w = 0$, then stagnation point. Streamlines only cross at stagnation points.

Particle Path:

Obviously:

$$\underline{u} = \frac{d\underline{x}}{dt}$$ - Lagrangian description.

So, expanding out the terms:

$$\frac{dx}{dt} = u \qquad \frac{dy}{dt} = v \qquad \frac{dz}{dt} = w$$ - the particle path equations.

If the velocity flow is NOT a function of time, then the streamlines & particle path equations are the same.

Proof:

$$u = \frac{dx}{dt} \qquad \Rightarrow \qquad \int_{x_0}^{x} \frac{dx}{u} = \int_{t_0}^{t} dt$$

Which can only be done if u is not a function of time.
Now:

$$\int_{t_0}^{t} dt = t - t_0$$

It follows that:

$$\int_{x_0}^{x} \frac{dx}{u} = \int_{y_0}^{y} \frac{dy}{v} = \int_{z_0}^{z} \frac{dz}{w} = t - t_0$$

Now, differentiate w.r.t x, say:

$$\frac{d}{dx} \int_{x_0}^{x} \frac{dx}{u} = \frac{1}{u} = \frac{d}{dx} \int_{y_0}^{y} \frac{dy}{v} = \frac{dy}{dx} . \frac{d}{dy} \int_{y_0}^{y} \frac{dy}{v}$$
$$= \frac{1}{v} . \frac{dy}{dx}$$
$$= \frac{1}{w} . \frac{dz}{dx}$$

That is:

$$\frac{1}{u} = \frac{1}{v}\frac{dy}{dx} = \frac{1}{w}\frac{dz}{dx} \qquad \Leftrightarrow \qquad \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Which is back to the streamline equation.