Method of Images

e.g.

$$w(z) = \frac{m}{2p}\log(z-a) + \frac{m}{2p}\log(z+a)$$

Is equivalent to a source at $z = a$, with a boundary on $x = 0$.

Mirror all sources/sinks/vortices in all “boundaries”

Remember that on streamlines:

$$y = \mathrm{Im}\{w(z)\} = const$$

Notation: $\bar{f}(z)$ conjugates the function, not *z*.

Circle theorem:

If $w = f(z)$ is a given flow, and if a circle is placed in the flow at the origin, with a radius $|z| = a$, then, under the assumption that *f(z)* is analytic inside & on the circle, the new potential is:

$$w(z) = f(z) + \bar{f}\left(\frac{a^2}{z}\right)$$

e.g.

1) flow due to a source m at $z = z_0$, around a circle $|z| = a$.

Source flow: $f(z) = \frac{m}{2p}\log(z - z_0)$

Conjugate function: $\bar{f}(z) = \frac{m}{2p}\log(z - \bar{z}_0)$

$$\therefore \quad \bar{f}\left(\frac{a^2}{z}\right) = \frac{m}{2p}\log\left(\frac{a^2}{z} - \bar{z}_0\right)$$

$$= \frac{m}{2p}\log\left(\frac{a^2 - z\bar{z}_0}{z}\right)$$

$$= \frac{m}{2p}\log\left(-\bar{z}_0\,\frac{-\frac{a^2}{\bar{z}_0} + z}{z}\right)$$

$$= \frac{m}{2p}\log\left(z - \frac{a^2}{\bar{z}_0}\right) - \frac{m}{2p}\log(z) + \underbrace{\frac{m}{2p}\log(-\bar{z}_0)}_{const}$$

Thus, the final complex potential is:

$$
\begin{aligned}
w(z) &= f(z) + \bar{f}\left(\frac{a^2}{z}\right) \\
&= \underbrace{\frac{m}{2\pi}\log(z - z_0)}_{\text{original source}} + \underbrace{\frac{m}{2\pi}\log\left(z - \frac{a^2}{\bar{z}_0}\right)}_{\text{image at } \frac{a^2}{\bar{z}_0}} - \underbrace{\frac{m}{2\pi}\log(z)}_{\text{image at origin}}
\end{aligned}
$$

2) Uniform flow past a circle, with $\alpha = 0$.

$$
f(z) = Uz \qquad \bar{f}(z) = Uz \qquad f\left(\frac{a^2}{z}\right) = \frac{Ua^2}{z}
$$

Thus: $$
\begin{aligned}
w(z) &= Uz + \frac{Ua^2}{z} \\
&= U\left(re^{i\theta} + \frac{a^2}{r}e^{-i\theta}\right)
\end{aligned}
$$

Thus, the streamlines are on:

$$
\psi = \operatorname{Im}\{w(z)\} = U\left(r\sin\theta - \frac{a^2}{r}\sin\theta\right) = U\sin\theta\left(r - \frac{a^2}{r}\right)
$$

What we can do with this now, is work out the velocity. In polars this is:

$$
\begin{aligned}
\underline{u} &= \left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}, -\frac{\partial\psi}{\partial r}\right) \\
&= \left(U\left(1 - \frac{a^2}{r^2}\right)\cos\theta, -U\left(1 + \frac{a^2}{r^2}\right)\sin\theta\right)
\end{aligned}
$$

From this, can find the velocity at certain points, e.g. on the circle. Can also find the stagnation points.