Uniform stream:

Speed U at angle a to the x-axis. Thus, the velocity vector is just:

$$\underline{u} = (U\cos a, U\sin a) = (u, v)$$

Now, the complex velocity is given by:

$$\frac{dw}{dz} = u - iv = U\cos a - iU\sin a = Ue^{-ia}$$

Thus:

$$\frac{dw}{dz} = Ue^{-ia} \Rightarrow dw = Ue^{-ia}\,dz$$

$$\therefore \quad \boxed{w(z) = Uze^{-ia}}$$

Source:

At any radius, mass flux must be the same. The velocity is also purely radial.

$$2prU = m \qquad m \text{ some constant}$$

$$\frac{dw}{dz} = u - iv = U\cos q - Ui\sin q = Ue^{-iq}$$

$$\therefore \quad \frac{dw}{dz} = \frac{U}{e^{iq}} = \frac{m}{2pre^{iq}} = \frac{m}{2pz}$$

$$\therefore \quad w(z) = \int \frac{m}{2pz}\,dz = \frac{m}{2p}\log(z)$$

Thus, for a source of strength m, at $z = z_0$, the complex potential is:

$$\boxed{w(z) = \frac{m}{2p}\log(z - z_0)} \qquad \text{if } m < 0\text{, then sink}$$

Vortex:

Here, the velocity is purely tangential. Again, the mass flux through any radius is constant, k say:

$$2prV = k$$

$$\underline{u} = V\hat{\underline{q}} = (-V\sin q, V\cos q)$$
$$\frac{dw}{dz} = u - iv = -iV(\cos q - i\sin q) = -iVe^{-iq}$$

$$\therefore \qquad \boxed{w(z) = -\frac{ik}{2p}\log(z - z_0)}$$

k +ve… anti-clockwise
k –ve… clockwise convention

Dipole:

Suppose a source & a sink very close together…

$$w(z) = \frac{m}{2p}\log\left(z - de^{ia}\right) - \frac{m}{2p}\log(z)$$
$$= \frac{m}{2p}\log\left(1 - \frac{de^{ia}}{z}\right)$$

Now, let $d \to 0$ & $dm \equiv m$ be constant.
Using the expansion:

$$\log(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$$

Get:

$$w(z) = \frac{m}{2p}\left(-\frac{de^{ia}}{z} - \frac{d^2e^{2ia}}{2z^2} - \dots\right)$$
$$= -\frac{m}{2p}\left(\frac{e^{ia}}{z} + \frac{de^{2ia}}{2z^2} + O(d^2)\right)$$

$\therefore$ as $d \to 0$:

$$\boxed{w(z) = -\frac{me^{ia}}{2p(z - z_0)}}$$

for a dipole at $z = z_0$, inclined by a

Flow in a corner:

$$\boxed{w(z) = Az^m = Ar^m e^{imq}}$$