Singularities:

$b_n \equiv$ principle part

Isolated singularities

- small neighbourhood of the point z_0 (a singularity of the function *f(z)*) which contains no other singularities;

- If $b_n = 0$ for all *n*:

  $z = z_0$ "removable singularity"

- If $b_n = 0$ for $n > k$: finite number of terms

  "pole of order *k*"

  If $k = 1$ ➔ simple pole
  If $k = 2$ ➔ double pole

  Order also given by the power on the bottom of the function

- If $b_n = 0$ has an infinite number of terms:

  "isolated essential singularity"

Examples:

$$f(z) = \frac{1}{1-z}$$ pole, order 1 at $z = 1$

$b_n = 0$ all *n*, except $n = 1$

$$f(z) = \frac{1}{(z-3i)^2}$$ pole, order 2 at $z = 3i$

$$f(z) = \frac{1}{(z-1)(z-2)}$$ two poles, both order 1 at $z = 2$ & $z = 1$

Non-isolated singularities

- in any small region surrounding a singularity, there is always another.

- Essential singularity.

  e.g. $f(z) = \dfrac{1}{\sin \frac{1}{z}}$ singular at $\frac{1}{z} = np \Rightarrow z = \dfrac{1}{np}$

- Branch point singularity.

  e.g. $f(z) = \ln z = \ln|z| + iArg(z)$

  cut plane – only uniquely defined here

  branch point is non-isolated here.. singular everywhere on branch point