Laplace Transforms:

$$L\{f(t)\} \equiv \bar{f}(p) \equiv \int_0^\infty f(t)e^{-pt}dt$$

The operator is linear. That is:

$$L\{f_1(t)+f_2(t)\} = L\{f_1(t)\} + L\{f_2(t)\}$$
$$L\{af(t)\} = aL\{f(t)\}$$

Assume the contribution from the infinite-limit is zero.
Some common transforms:

$f(t)=1$	$\bar{f}(p)=\dfrac{1}{p}$
$f(t)=e^{at}$	$\bar{f}(p)=\dfrac{1}{p-a}$
$f(t)=t^n$	$\bar{f}(p)=\dfrac{n!}{p^{n+1}}$
$f(t)=t^n e^{at}$	$\bar{f}(p)=\dfrac{n!}{(p-a)^{n+1}}$

$$L\left\{\frac{dy}{dt}\right\} = \int_0^\infty \frac{dy}{dt}e^{-pt}dt$$
$$= \left[ye^{-pt}\right]_0^\infty + p\int_0^\infty ye^{-pt}dt$$
$$= -y(0) + p\bar{y}(p)$$
$$L\left\{\frac{d^2y}{dt^2}\right\} = \int_0^\infty \frac{d^2y}{dt^2}e^{-pt}dt$$
$$= \left[\frac{dy}{dt}e^{-pt}\right]_0^\infty - \int_0^\infty\left(-p\frac{dy}{dt}e^{-pt}\right)dt$$
$$= -y'(0) - py(0) + p^2\bar{y}(p)$$

Thus:

$$L\left\{\frac{dy}{dt}\right\} = p\bar{y}(p) - y(0)$$

$$L\left\{\frac{d^2y}{dt^2}\right\} = p^2\bar{y}(p) - y'(0) - py(0)$$

$$
\begin{aligned}
L\{e^{iwt}\} &= \int_0^{\infty} e^{iwt} e^{-pt}\,dt \\
&= \int_0^{\infty} e^{t(iw-p)}\,dt \\
&= -\frac{1}{iw-p} = \frac{1}{p-iw}\cdot\frac{p+iw}{p+iw} \\
&= \frac{p+iw}{p^2+w^2}
\end{aligned}
$$

Thus, from this result, can see that:

$$
\begin{aligned}
L\{\cos wt\} &= L\{\mathrm{Re}\{e^{iwt}\}\} = \mathrm{Re}\{L\{e^{iwt}\}\} \\
&= \frac{p}{p^2+w^2}
\end{aligned}
$$

And $L\{\sin wt\} = \dfrac{w}{p^2+w^2}$

To invert, if we cannot do by inspection, use the inversion integral:

$$
f(t) = L^{-1}\{\bar{f}(p)\} = \frac{1}{2pi}\int_{c-i\infty}^{c+iw} \bar{f}(p) e^{pt}\,dp
$$

Where c is chosen to the right of all singularities of $\bar{f}(p)e^{pt}$.
c.f. Jordan's Lemma.
Thus, only get contributions to integral from singularities & their residues:

$$
f(t) = \frac{1}{2pi}\left[2pi \sum_{\text{to left of c}} \mathrm{Res}\{\bar{f}(p)e^{pt}\}\right]
$$

We can use Laplace transforms to solve differential equations:

$$
\frac{dy}{dt} + y = e^{-2t} \qquad y(0) = 1
$$

Take LT of whole thing, remembering that it is linear:

$$
L\left\{\frac{dy}{dt}\right\} + L\{y(t)\} = L\{e^{-2t}\}
$$

Or:

$$
p\bar{y}(p) - \underbrace{y(0)}_{=0} + \bar{y}(p) = \frac{1}{p+2} \qquad \text{making use of initial conditions}
$$

$$
\bar{y}(p) = \frac{p+3}{(p+2)(p+1)} = \frac{2}{p+1} - \frac{1}{p+2}
$$

Which can be inverted by inspection:

$$y(t) = 2e^{-t} - e^{-2t}$$

Function	Laplace Transform
e^{at}	$\dfrac{1}{p-a}$
t^n	$\dfrac{n!}{p^{n+1}}$
$t^n e^{at}$	$\dfrac{n!}{(p-a)^{n+1}}$
$\cos wt$	$\dfrac{p}{p^2+w^2}$
$\sin wt$	$\dfrac{w}{p^2+w^2}$
$\dfrac{dy}{dt}$	$p\bar{y}(p) - y(0)$
$\dfrac{d^2y}{dt^2}$	$p^2\bar{y}(p) - y'(0) - py(0)$