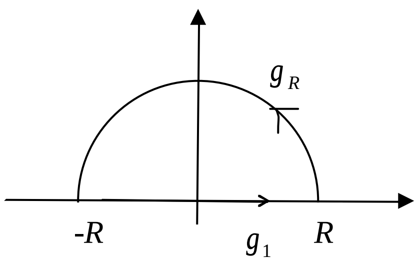
Jordan's lemma:

Consider the integral:



$$\oint_{g} f(z)e^{imz}dz$$

Where:

$$g = g_1 + g_R$$

Thus:

$$\oint_{g} f(z)e^{imz}dz = \int_{g_1} f(z)e^{imz}dz + \int_{g_R} f(z)e^{imz}dz$$
$$= \int_{-R}^{R} f(z)e^{imz}dz + \int_{g_R} f(z)e^{imz}dz$$

Then, *if the following conditions hold*:

$$m > 0$$
$$\lim_{z\to 0}\left|f(z)\right| = 0$$
$f(z)e^{imz}$ has only isolated, finite singularities

Then, as $R \to \infty$, that is to say, the semi-circular contour closes at infinity, then:

$$\oint_{g} f(z)e^{imz}dz = \lim_{R\to 0}\left\{\int_{-R}^{R} f(z)e^{imz}dz + \int_{g_R} f(z)e^{imz}dz\right\} = \int_{-\infty}^{\infty} f(z)e^{imz}dz$$

Notice that the semi-circular integral goes to zero. Hence:

$$\oint_{g} f(z)e^{imz}dz = \int_{-\infty}^{\infty} f(z)e^{imz}dz = \int_{-\infty}^{\infty} f(x)e^{imx}dz$$

Which is just an integral along the real line.
Therefore, we can use this to evaluate real integrals by *Cauchy's residue theorem*:

$$\int_{-\infty}^{\infty} f(x)e^{imx}dz = \oint_{g} f(z)e^{imz}dz = \frac{1}{2pi}\sum_{k}\text{Res}\left\{f(z)e^{imz}; z = z_k\right\}$$

Notice that it is the sum over residues which are enclosed by the contour. Which is saying: sum over the residues due to singularities in the upper-half-plane.

This is all provided that the above conditions hold!