

Integration:

$$\int_{\mathbf{g}} f(z) dz = \int_{t=a}^{t=b} f\{z(t)\} \frac{dz}{dt} dt$$

Cauchy's Theorem:

Jordan contour: contour with no intersections.

Let \mathbf{g} be a close Jordan contour in a domain D , and let $f(z)$ be a regular function inside \mathbf{g} , and continuous on \mathbf{g} . (i.e. \mathbf{g} does not enclose any singularities).

Then:

$$\oint_{\mathbf{g}} f(z) dz = 0 \quad \text{regular - thus Cauchy-Riemann holds.}$$

Green's Theorem:

$$\int_{\mathbf{g}} P dx + Q dy = \int_A \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right] dx dy \quad \text{- use to "prove" Cauchy's Theorem.}$$

Corollary 1:

$$\int_{\mathbf{g}} f(z) dz = \int_{\tilde{\mathbf{g}}} f(z) dz$$

Independent of path. Only dependant upon end points.

Corollary 2:

Let $f(z)$ be regular in D . Let z_1, z_2 be elements of D .

Define:

$$F(z) = \int_{z_1}^{z_2} f(\mathbf{z}) d\mathbf{z}$$

Then:

- (i) $F(z)$ also regular in D ;
- (ii) $F'(z) = f(z)$;
- (iii) $\int_{z_1}^{z_2} f(\mathbf{z}) d\mathbf{z} = \int_{z_1}^{z_2} F'(\mathbf{z}) d\mathbf{z} = F(z_1) - F(z_2)$

Hence, normal rules of integration apply.

Corollary 3:

Let $f(z)$ be regular in a region bounded by two closed Jordan contours $\mathbf{g}_1, \mathbf{g}_2$. \mathbf{g}_1 inside \mathbf{g}_2 . Then:

$$\oint_{\mathbf{g}_1} f(z) dz = \oint_{\mathbf{g}_2} f(z) dz$$

- can deform/shrink contours.

All of above D is a simply connected domain.