Basics:

$\cosh(iz) = \cos z$

$\sinh(iz) = i\sin z$ $\qquad z = x + iy$

$\ln(z) = \ln r + iq$

Principal value range: range within which function is well defined/single valued.

OPEN: in a set of complex numbers, if one can draw an arbitrarily small circle around any point, with the circle still within the set, then set is open.

CONNECTED: if two points in a set can be connected with a continuous curve, then set is connected.

If the set is not-empty & open & connected, then the set is a "domain".

Continuity:

If at $z = z_0$: $\quad |f(z) - f(z_0)| \to 0 \quad$ as $\quad |z - z_0| \to 0$

In any manner,

Then function is continuous.

Differentiability:

Differentiable at $z = z_0$ if:

$$\lim_{z\to z_0}\left(\frac{f(z)-f(z_0)}{z-z_0}\right) = \left.\frac{df}{dz}\right|_{z=z_0}$$

Exists, and is unique as $|z - z_0| \to 0$ in any manner.

Differentiability implies continuity, but continuity does not imply differentiability.

Regular if differentiable everywhere. Where not differentiable called a singularity.

Cauchy-Riemann:

A function: $\quad f(z) = u(x, y) + iv(x, y)$

Is regular if and only if

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

hold everywhere in its domain of definition

u & *v* are conjugate function… and are harmonics. That is, they satisfy:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$