Nuclei:

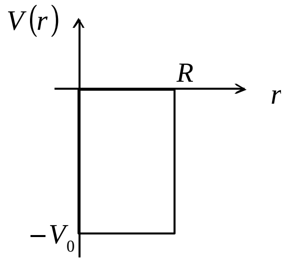
Nucleus… radius ~1fm = $10^{-15} m$

Isotopes … same Z, different A
Isobars … same A, different Z

Z = number of protons A = number of protons + neutrons

Proton = uud
Neutron = ddu (quarks)

Deuteron:



(1) … inside nucleus… $r < R$
(2) … outside nucleus… $r > R$

Can write the Schrödinger equation:

$$\nabla^2 u(r) + \frac{2m_r}{\hbar^2}(E - V(r))u(r) = 0$$

Thus, if spherical symmetry:

$$\frac{d^2}{dr^2}u(r) + \frac{2m_r}{\hbar^2}(E - V(r))u(r) = 0$$

m_r = reduced mass

Assume no relative angular momentum

(1) $r < R$ $V(r) = -V_0$, $E = -E_B$ (binding energy)

$$\therefore \quad \frac{d^2u}{dr^2} + \frac{2m_r}{\hbar^2}(V_0 - E_B)u = 0$$

(2) $r > R$ $V(r) = 0$, $E = -E_B$

$$\therefore \quad \frac{d^2u}{dr^2} - \frac{2m_r}{\hbar^2}(E_B)u = 0$$

Now, the reduced mass can be simplified:

$$m_r = \frac{m_p m_n}{m_p + m_n} \approx \frac{m_n m_n}{m_n + m_n} = \frac{m_n}{2}$$

Now, a solution to the TISE is of the form:

$$u(r) = Ae^{ikr} + Be^{-ikr}$$

Thus, for (1) & (2):

$$k_1 = \frac{1}{\hbar}\sqrt{m_n(V_0 - E_B)}$$
$$k_2 = \frac{i}{\hbar}\sqrt{m_n E_B}$$

Now, the boundary conditions:

$$u(0) = 0 \qquad\qquad \lim_{r\to\infty} u(r) = 0$$

Hence, applying to (1):

$$u_1(0) = A_1 + B_1 = 0 \Rightarrow A_1 = -B_1$$
$$\therefore \quad u_1(r) = A_1\left(e^{ik_1 r} - e^{-ik_2 r}\right)$$
$$= 2iA_1 \sin(k_1 r)$$

And to (2):

$$A = 0$$
$$\therefore \quad u_2(r) = B_2 e^{-ik_2 r}$$

Also, the wavefunction must be continuous at $r = R$:

$$\Rightarrow \quad u_1(R) = u_2(R) \qquad\qquad \left.\frac{du_1}{dr}\right|_R = \left.\frac{du_2}{dr}\right|_R$$

$$\Rightarrow \quad 2iA_1 \sin(k_1 R) = B_2 e^{-ik_2 R} \qquad\qquad 2iA_1 k_1 \cos(k_1 R) = ik_2 B_2 e^{-ik_2 R}$$

Dividing the two expressions results:

$$k_1 \cot(k_1 R) = ik_2$$

We can measure E_B by firing a photon at a deuteron, and the minimum energy which will split the deuteron is the binding energy:

$$g + {}_1^2 D \rightarrow {}_1^1 H + {}_0^1 n$$

Liquid drop model:

$$M(Z,A) = Zm_H + Nm_n - B(Z,A)/c^2$$

If the system is bound:

$$B(Z,A) > 0$$

Know the shape of the B/A – A graph… which is approx constant on 8MeV, with a max at *A = 56*.

Semi-Empirical Mass formula:

Volume term: $B_V = a_V A$

Coulomb term: $B_C = -a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}}$

As $R \propto A^{\frac{1}{3}}$ … repulsive… *Z(Z-1)* proton pairs

Surface term: $B_S = -a_S A^{\frac{2}{3}}$

Surface area $\propto R^2 \propto \left(A^{\frac{1}{3}}\right)^2$

Asymmetry term: $B_a = -a_a \frac{\left(Z - \frac{A}{2}\right)^2}{A}$

Keeps difference between number of protons & neutrons small

Stability term: $d = -a_p A^{-\frac{1}{2}}$

Describes if odd/even for *A*/*Z*

So, in total, the binding energy can be written:

$$B(Z,A) = a_V A - a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_S A^{\frac{2}{3}} - a_a \frac{\left(Z - \frac{A}{2}\right)^2}{A} + d$$

Thus, the mass can be written:

$$M(Z,A) = Zm_H + Nm_n - a_V A + a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}} + a_S A^{\frac{2}{3}} + a_a \frac{\left(Z - \frac{A}{2}\right)^2}{A} - d$$

Stable nuclei:

Most bound nuclei for a given *A* must have a max. binding energy:

$$\left(\frac{\partial B}{\partial Z}\right)_A = 0$$

Thus:

$$\left(\frac{\partial B}{\partial Z}\right)_A = -a_C \frac{2Z-1}{A^{\frac{1}{3}}} - 2a_a \frac{\left(Z - \frac{A}{2}\right)}{A} = 0$$

$$\Rightarrow \qquad -a_C \frac{2Z-1}{A^{\frac{1}{3}}} = 2a_a \frac{\left(Z - \frac{A}{2}\right)}{A}$$

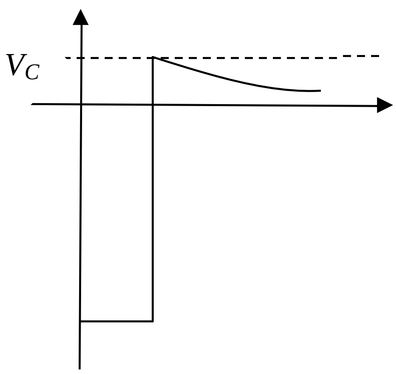
Can solve this, with $2Z-1 \approx 2Z$ … to get the most stable Z for a given A… Z_0.
Particles will decay down to Z_0:

$Z < Z_0$ b^- decay $n \to p + b^- + \overline{u}_e$

$Z > Z_0$ b^+ decay $p \to n + b^+ + n_e$

Probing nuclear structure:

In Rutherford scattering, alpha particles are incident upon thin gold foil.
Alpha interacts with both strong & coulomb forces.



$$V_C = \frac{Z_a Z_{Au} e^2}{4 p e_0 r}$$ … coulomb barrier

$b \propto \cot\frac{q}{2}$ … where q is the scattering angle & b the impact parameter: the distance from the centre of the nucleus in which the particle impacts. Hence can infer nuclear size from this.

The energy of the incident alpha is E_a. Thus, if $E_a > V_C$, the coulomb barrier is overcome and the alpha is absorbed. This energy can be found, and thus the radius of the nucleus found. This also implies however, that we cannot use a -particles to probe nuclear structure.

Electrons are not affected by the strong nuclear force… which is the force which "sucks" the alpha particles in… so can use e^- to probe nuclear structure.

$$E_e = pc \qquad l = \frac{h}{p} = \frac{hc}{E_e}$$

Thus, the minimum distance which an electron can resolve is given by its de Broglie wavelength.
To increase the resolution (thus decrease minimum distance scales) need to increase the incident electron-energy.

Now, if this data is plotted, a straight line is found, and:

$$r = r_0 A^{\frac{1}{3}} \qquad r_0 = 1.2\,fm$$

Radioactive decays:

Alpha… beta… fission… gamma

Decay law:

$$-\frac{dN}{dt} = lN$$

N = #radioactive nuclei

l =decay constant

Thus, solving gives:

$$N(t) = N_0 e^{-lt}$$

Half life… time when $N(t) = \frac{N_0}{2}$:

$$t_{\frac{1}{2}} = \frac{\ln 2}{l}$$

Average time nuclei survives before decay:

$$t = \frac{\int_0^\infty t\left|\frac{dN}{dt}\right|dt}{\int_0^\infty \left|\frac{dN}{dt}\right|dt} = \frac{1}{l}$$

Activity… decays per second:

$$A = -\frac{dN}{dt} = lN \qquad \Rightarrow \qquad A = lN$$

3 main sources of natural activity:

Primordial… before earth;
Cosmogenic… cosmic ray interactions;
Human origin… fires/fission/reactors etc…

Dating:

If a decay happens:

$$P \rightarrow D$$

Then:

$$N_P(t=0) = N_P(t) + N_D(t)$$

$$\therefore \quad N_P(t) = N_p(t=0)e^{-lt} = [N_P(t) + N_D(t)]e^{-lt}$$

Thus, as every quantity can be measured, solve for t to date things!

Carbon dating:

^{12}C … stable $\qquad$ ^{14}C … unstable

Thus, can say:

$$N\left(^{14}C\right) = N_0\left(^{14}C\right)e^{-lt}$$
$$N\left(^{12}C\right) = N_0\left(^{12}C\right)$$

Hence, the ratio can be measured:

$$\frac{N\left(^{14}C\right)}{N\left(^{12}C\right)} = \underbrace{\frac{N_0\left(^{14}C\right)}{N_0\left(^{12}C\right)}}_{\Pi} e^{-lt}$$

Where Π is assumed to be constant. This is only true up until ~1900, as fossil fuels & such like have messed up this ratio.

Alpha decay:

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \underbrace{^{4}_{2}He^{++}}_{\equiv a}$$

Now, if X initially at rest, by energy conservation:

$$M_X c^2 = M_Y c^2 + E_Y + M_a c^2 + E_a$$

Thus, rearranging, and defining the Q-value:

$$T_Y + T_a = (M_X - M_Y - M_a)c^2 \equiv Q$$

Now, a -decay is only possible for $Q > 0$.
Also, note that:

$$Q = B_a + B_Y - B_X \qquad \text{binding energies…}$$

To find out how much KE the a has… i.e. T_a ; consider momentum conservation:

$$p_a = p_Y \qquad \Rightarrow \qquad p_a^2 = p_Y^2$$

Kinetic energies are:

$$T_a = \frac{p_a^2}{2m_a} \qquad \qquad p_a^2 = 2T_a m_a$$
$$\Rightarrow$$
$$T_Y = \frac{p_Y^2}{2m_Y} \qquad \qquad p_Y^2 = 2T_Y m_Y$$

Thus:

$$1 = \frac{T_a m_a}{Y_Y m_Y} \qquad \Rightarrow \qquad T_Y = \frac{m_a}{m_Y} T_a$$

$$Q = T_a + T_y = T_a + \frac{m_a}{m_Y T_a}$$

Hence, rearranging:

$$T_a = \frac{Q}{1 + \frac{m_a}{m_Y}} \qquad \text{the kinetic energy of the resulting } a \text{ -particles.}$$

$$E_a \approx 50 MeV$$

Fission:

$$A \rightarrow X + Y$$

Symmetric fission is of the form:

$$A \rightarrow 2X$$

Thus, the energy released from fission:

$$E_F / c^2 = M(A, Z) - 2M\left(\frac{A}{2}, \frac{Z}{2}\right)$$

$E_F > 0$ for fission to happen, and if the SEMF is looked at, all but the surface & coulomb terms cancel; leaving:

$$E_F = -0.26 a_S A^{\frac{2}{3}} + 0.37 a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

Which corresponds to:

$\frac{Z^2}{A} \geq 0$ should spontaneously fissure

But it does not…

… this is due to the activation energy which is needed to be overcome as atom deforms from being spherical, to more ellipsoidal.

$E_F \approx 200 MeV$