Basic Relations:

Relativistic Energy/Momentum:

$$E^2 = p^2c^2 + m^2c^4$$

For photons:

$$E = pc$$
$$E = hn$$

Hence:

$$p = \frac{h}{l} \Rightarrow l = \frac{h}{p}$$

Non-relativistic:

$$E = \frac{p^2}{2m}$$ The KE

Hence:

$$p = \sqrt{2mE}$$
$$l = \frac{h}{\sqrt{2mE}}$$

Constructive interference in diffraction if:

$$d \sin q = n l$$

The Heisenberg uncertainty principle reads:

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \hbar = \frac{h}{2p}$$

Now, $E = hn$ and $w = 2pn$. Therefore:

$$E = \hbar w$$

Similarly:

$$k = \frac{2p}{l} = \frac{2pp}{h} = \frac{p}{\hbar} \Rightarrow p = \hbar k$$

Hence, as $E = \frac{p^2}{2m}$, then:

$$\hbar w = \frac{\hbar^2 k^2}{2m}$$

The Schrödinger Equation:

The time dependant Schrödinger equation reads:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

Suppose a solution is of the separable form:

$$\Psi(x,t) = y\ (x)T(t)$$

Then:

$$\frac{\partial^2 \Psi}{\partial x^2} = T\frac{d^2 y}{dx^2} \qquad \frac{\partial \Psi}{\partial t} = y\ \frac{dT}{dt}$$

Hence, the TDSE becomes:

$$-\frac{\hbar^2}{2m}T\frac{d^2 y}{dx^2} + V(x)yT = i\hbar y\ \frac{dT}{dt}$$

Dividing by $\Psi = yT$:

$$-\frac{\hbar^2}{2my}\frac{d^2 y}{dx^2} + V(x) = \frac{i\hbar}{T}\frac{dT}{dt}$$

Which is only true if both sides are equal to a constant, E:

$$-\frac{\hbar^2}{2my}\frac{d^2 y}{dx^2} + V(x) = E$$

AND: $\frac{i\hbar}{T}\frac{dT}{dt} = E$

The "time equation" can be solved for T:

$$\frac{1}{T}dT = \frac{E}{i\hbar}dt \Rightarrow \ln T = \frac{E}{i\hbar}t \Rightarrow T(t) = e^{\frac{Et}{i\hbar}} = e^{-\frac{iEt}{\hbar}} = e^{-iwt}$$

And the "space equation" becomes the time independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2 y}{dx^2} + V(x)y\ = Ey$$

And (notation) using operators this compresses to:

$$\hat{H}y = Ey$$

Where:

$$\hat{H} = \frac{\hat{p}_x^{\ 2}}{2m} + V(\hat{x})$$

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x} \qquad \hat{T} = \frac{\hat{p}_x^2}{2m}$$

$$\hat{x} = x \qquad \therefore \hat{H} = \hat{T} + V(\hat{x})$$

Now, expectation values can be computed:

$$< p >= \int_{-\infty}^{+\infty} \Psi^* \hat{p}\Psi dx \qquad < p^2 >= \int_{-\infty}^{+\infty} \Psi^* \hat{p}^2\Psi dx$$

$$< x >= \int_{-\infty}^{+\infty} \Psi^* \hat{x}\Psi dx \qquad < x^2 >= \int_{-\infty}^{+\infty} \Psi^* \hat{x}^2\Psi dx$$

Where Ψ^* is the complex conjugate of Ψ , which is usually a complex function.
Hence the uncertainties can be found:

$$(\Delta x)^2 =< x^2 > - < x >^2$$

$$(\Delta p)^2 =< p^2 > - < p >^2$$

Commutivity:

$$[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \prec \begin{cases} = 0 & \text{comutate} \\ \neq 0 & \text{don't comutate} \end{cases}$$

1D Potential Wells:

V = 0 – in some cases, depends upon initial conditions.

Hence, TISE becomes:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 y}{\partial x^2} = Ey$$

Or:

$$\frac{\hbar^2}{2m}\frac{\partial^2 y}{\partial x^2} + Ey = 0$$

Or:

$$\frac{\partial^2 y}{\partial x^2} + k^2 y = 0$$

With:

$$k^2 = \frac{2mE}{\hbar^2} \qquad \Rightarrow \qquad E = \frac{k^2\hbar^2}{2m}$$

The solutions to the TISE $\hat{H}y = Ey$ give eigenfunctions & eigenvalues:

$$\hat{H}y_n = E_n y_n$$