Angular Momentum Operators & Commutators:

Classically: $\underline{L} = \underline{r} \times \underline{p}$

Hence, quantum mechanically: $\hat{\underline{L}} = \hat{\underline{r}} \times \hat{\underline{p}}$

Now:
$$\hat{\underline{p}} = -i\hbar\nabla$$
$$\hat{\underline{r}} = \hat{x}\underline{i} + \hat{y}\underline{j} + \hat{z}\underline{k}$$

Hence:

$$\hat{\underline{L}} = -i\hbar \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x & y & z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix}$$

Thus:

$$\hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \right)$$
$$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z} \right)$$
$$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \right)$$

And:

$$\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$$
$$\left[\hat{L}_y, \hat{L}_z\right] = i\hbar\hat{L}_x$$
$$\left[\hat{L}_z, \hat{L}_x\right] = i\hbar\hat{L}_y$$
$$\left[\hat{H}, \hat{L}_z\right] = 0$$

Remembering that: $\left[\hat{A}, \hat{B}\right] = -\left[\hat{B}, \hat{A}\right]$

Also: $\hat{\underline{L}}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$

Hence:

$$\left[\hat{\underline{L}}^2, \hat{L}_x\right] = \left[\hat{\underline{L}}^2, \hat{L}_y\right] = \left[\hat{\underline{L}}^2, \hat{L}_z\right] = 0$$