

## Calculus of Variations:

$$I\{y(x)\} = \int_{x_1}^{x_2} f(x, y, y') dx \quad \text{Where } y' = \frac{dy}{dx}$$

Gives the Euler equation:

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0 \quad - \text{The extremum condition.}$$

Or, in another form:

$$f - y' \frac{df}{dy'} = 0 \quad \text{if } f \text{ doesn't depend on } x.$$

See that Euler's equation is like Lagrange's equation:

$$\frac{\partial f}{\partial y} = \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) \leftrightarrow \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i}$$

## Principle of Least Action:

Now:  $L = T - V$

$$L(q_i, \dot{q}_i, t)$$

For a moving body:

$$\text{Initial: } t = t_1 \quad q_i(t_1) = q_i^1$$

$$\text{Final: } t = t_2 \quad q_i(t_2) = q_i^2$$

Now, want to find a path that will minimise the action, A:

$$A = \int_{t_1}^{t_2} L(q_i, \dot{q}_i, t) dt = \int_{t_1}^{t_2} L dt$$

$$A\{q_i(t)\}$$

Which gives the action as the Lagrange equation, but found from Euler's equation for least action:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i}$$

To minimise A, find the derivatives wrt to the amplitudes involved, and solve.