Magnetostatics:

Lorentz Force Law:

$$\underline{F} = q\left(\underline{E} + \underline{v} \times \underline{B}\right)$$

If $\underline{E} = 0 \Rightarrow \underline{F} = q\underline{v} \times \underline{B}$

Motion of a charge in $\underline{B}$-field:
$\underline{F}$ perpendicular to velocity, hence motion in a circle.

$$F = \frac{mv^2}{r} = qvB$$

Period:

$$t = \frac{2pr}{v} = \frac{2pm}{qB}$$

Hence, the "cyclotron frequency" is:

$$u = \frac{qB}{2pm}$$

Maxwell's 2nd equation:

$$\nabla \cdot \underline{B} = 0$$

Current density is the number of charges, times the charge, times the drift velocity of the charges. Alternatively, it is the current per unit area.

$$j = \frac{I}{A} = -Nev_d$$

Alternatively, in integral form:

$$I = \int_S \underline{j} \cdot d\,\underline{A}$$

Ohm's law becomes:

$$\underline{j} = s\ \underline{E}$$

Where s is the electrical conductivity.

Hall Effect:

Current $\underline{I}$ flows along a slab of conductor, with width w and depth d – hence an area A. A magnetic field $\underline{B}$ intersects the slab, so that $\underline{B}$ is perpendicular to $\underline{I}$. Hence, the charge carriers "feel" a force. The charge carriers are electrons, with a drift velocity v_d.

$$F_m = \text{Magnetic force on –ve charge carriers}$$
$$F_e = \text{Electric force on charge build-up}$$

Now:

$$F_m = qvN = ev_d B$$

Also, the current is:

$$I = nev_d A \Rightarrow v_d = I/neA$$

Hence:

$$F_m = \frac{eIB}{neA}$$

To find the potential set up at equilibrium… i.e. that: $F_m = F_e$.
Where:

$$F_e = eE = \frac{V_H e}{w}$$

Hence, system in equilibrium when:

$$\frac{V_H e}{w} = \frac{eIB}{neA}$$

Therefore, the potential set up, the "Hall Voltage", which is across the slab, is:

$$V_H = \frac{IB}{ned}$$

Force on a Wire:

Force on one electron:

$$\underline{f} = -e\underline{v} \times \underline{B}$$

If the conductor has length $d\underline{l}$, and area $\underline{A}$… $nAdl$ electrons, at velocity v_d. Hence, the total force:

$$\underline{F} = -(nAdl)e\underline{v}_d \times \underline{B}$$

But, $I = -neAv_d$. Hence;

$$\underline{F} = Id\underline{l} \times \underline{B}$$

Ampere's Law:

$$\oint \underline{B} \cdot d\underline{l} = m_0 \sum I$$

Used on a long, straight wire:

$$\oint B \cdot dl = 2prB = m_0 I$$

$$\Rightarrow B = \frac{m_0 I}{2pr}$$

Consider two // currents I_1 & I_2, separated by d:
Magnetic field due to I_1 :

$$B = \frac{m_0 I_1}{2pr}$$

Hence, force on I_2:

$$\underline{F} = I_2 d\underline{l} \times \underline{B}$$

Hence, finding force per unit length, just do the integrals:

$$F = \frac{m_0 I_1 I_2}{2pd}$$

Differential form of Ampere's Law:

$$\nabla \times \underline{B} = m_0 \underline{j}$$ Static case only!

Magnetic Vector Potential:

Has the following properties:

$$\underline{A}$$

$$\underline{B} = \nabla \times \underline{A}$$

$$\nabla \cdot \underline{A} = 0$$

$$\nabla^2 \underline{A} = -m_0 \underline{j}$$

$$\underline{A} = \frac{m_0 I}{4p} \int \frac{d\underline{l}}{r} \Rightarrow \underline{A} // d\underline{l}$$