Magnetostatics:

Lorentz force law:

$$\underline{F} = q\left(\underline{E} + \underline{v} \times \underline{B}\right)$$

Motion of a charge in magnetic field, with everything at right angles:

$$F = \frac{mv^2}{r} = qvB$$

Period:

$$t = \frac{2pr}{v} = \frac{2pm}{qB}$$

Thus, cyclotron frequency is:

$$n = \frac{qB}{2pm}$$

Maxwell's 2nd Equation:

$$\boxed{\nabla \cdot \underline{B} = 0}$$

Current:

$$I = Nev_d A$$

Ohm's Law:

$$\boxed{\underline{j} = s\,\underline{E}}$$

Force on a wire:

$$\underline{F} = Id\underline{l} \times \underline{B}$$

$$d\underline{B} = m_0 I \frac{d\underline{l} \times \hat{\underline{r}}}{4pr^2}$$

Which is from the Biot-Savart law… gives the $\underline{B}$-field from a wire carrying a current.

Ampere's Law:

$$\oint_l \underline{B} \cdot d\underline{l} = m_0 \sum I$$

Or:

$$\boxed{\nabla \times \underline{B} = m_0 \underline{j}}$$ static case only!!

Magnetic Vector Potential:

$$\boxed{\underline{B} = \nabla \times \underline{A}} \qquad \underline{E} = -\frac{\partial \underline{A}}{\partial t} - \nabla \phi$$

And:

$$\underline{A} = \frac{\mu_0 I}{4\pi} \int \frac{d\underline{l}}{r}$$

a form of the Biot-Savart law

Magnetostatics in Materials:

Inductance, in vacuum, is:

$$L_0 = \mu_0 N^2 \pi r^2 l$$

And in materials is:

$$L = \mu L_0 \qquad \mu = \frac{1}{1 - \chi_B}$$

Magnetic dipoles:

Magnetic moment = current X area

$$\boxed{\underline{m} = I\,\underline{A}}$$

Torque on current loop:

$$\underline{\tau} = \underline{m} \times \underline{B}$$

Potential energy of loop:

$$U(\theta) = -\underline{m} \cdot \underline{B}$$

Magnetisation is like polarisation: total dipole moment per unit area:

$$\underline{M} = N\underline{m}$$

unit: amp.m^{-1}

Thus:

$$\boxed{\underline{M} = \chi_B \frac{\underline{B}}{\mu_0}}$$

χ_B = magnetic susceptibility

Forms of Magnetisation:

Diamagnetism: from individual atoms, and electrons orbiting in $\underline{B}$-fields
Paramagnetism: moments aligning
Ferromagnetism: due to domains within material

Hysterisis: in Ferro-magnets: process by which the material retains its magnetisation after external field removed.

Dia	Weak	Linear	-ve
Para	Stronger	Linear	+ve
Ferro	Strong	Non-linear	Permanent

Magnetic dipoles in materials can be visualised as small current loops, where the internal components cancel out, to leave only a surface current.

$$\underline{i}_s = \underline{M} \times \hat{\underline{n}} \qquad \underline{i}_b = \nabla \times \underline{M} \qquad \Rightarrow \underline{i} = \underline{i}_s + \underline{i}_b$$
$$\underline{B} = \underline{B}_0 + m_0 \underline{M}$$

Thus, introduce the magnetic field vector:

$$\underline{H} = \frac{\underline{B}}{m_0} - \underline{M} \qquad \Rightarrow \qquad \underline{B} = m m_0 \underline{H}$$

Thus:

$$\nabla \times \underline{H} = \underline{j}_f \qquad \text{“amperes law”}$$
$$\oint \underline{H} \cdot d\underline{l} = I_f$$

Energy in magnetic fields is thus:

$$U = \frac{1}{2} \underline{B} \cdot \underline{H}$$

For non-ferromagnetic materials $m \sim 1$
$\underline{H}$-field lines can be discontinuous:

$$\nabla \cdot \underline{H} = -\nabla \cdot \underline{M}$$

At boundaries:

B_{perp} continuous
$H_{//}$ continuous