Coulomb's Law:

$$\underline{F} = \frac{q_1 q_2}{4 p e_0 r^2} \hat{r}$$

Electric Potential:

From: $W = q \int_A^B \underline{E} \cdot d\underline{l}$

Potential difference:

$$f(r_a) - f(r_b) = -\int_a^b \underline{E} \cdot d\underline{l}$$

Potential in going from infinity to r:

$$f(r) = \frac{q}{4 p e_0 r}$$

$$\underline{E} = -\nabla f$$

In electrostatic fields *only*: $\nabla \times \underline{E} = 0$

Thus $\underline{E}$ is conservative.

Electric Dipole:

$$f(x,y,z) = \frac{1}{4 p e_0} \left(\frac{q}{\sqrt{x^2 + y^2 + (z-d)^2}} + \frac{-q}{\sqrt{x^2 + y^2 + (z+d)^2}} \right)$$

For the off-axis potential, for two charges –q and +q, separated by d.

Gauss' Law:

Integral form:

$$\oint_S \underline{E} \cdot d\underline{S} = \frac{1}{e_0} Q_{enc} = \frac{1}{e_0} \int_V r \, dV$$

Differential form:

$$\nabla \cdot \underline{E} = \frac{r}{e_0}$$ - Maxwell's 1st equation

Thus:

$$\nabla^2 f = \frac{r}{e_0}$$ - Poisson's equation

If no charge densities:

$$\nabla^2 f = 0$$ - Laplace's equation

Method of Images:

Same boundary conditions => same solution

Capacitors & Dielectrics:

$$C = \frac{Q}{V}$$

Find V from $\int_a^b \underline{E} \cdot d\underline{l}$

A dielectric material is an insulator: thus conductivity is zero.

Dielectrics gain dipole moments when placed in an electric field.

i.e. they polarise

$$C = eC_0$$

As dielectric inserted, capacitance increases.

Thus reduces the internal electric field.

Mechanisms for polarisation:

Electronic:

The atoms within the dielectric shift, their electron clouds moving, hence leaving the atom with a dipole… which then align with the applied electric field.

Aligned dipoles:

Works in a similar way, but this time the intrinsic dipoles of the molecules align – when there is no external field present, they are at "random".

Dipole moment:

$$\underline{p} = q\underline{d}$$

Polarisation: $\underline{P} = n\,\underline{p}$ like the total number of dipole moments.

Surface charge density:

$$s_p = \underline{P} \cdot \underline{\hat{n}}$$

Where $\underline{\hat{n}}$ is a normal to the surface.

$$\underline{P} = (e-1)e_0 \underline{E}$$
$$c_E = e - 1$$
$$r_p = -\nabla \cdot \underline{P}$$

In the function $\underline{P} = (e-1)e_0 \underline{E}$, we assumed:

Linear, isotropic, homogeneous & non-conducting

Electric Displacement Vector:

$$\underline{D} = e_0 \underline{E} + \underline{P}$$

Hence, a new version of Gauss' law:

$$\nabla \cdot \underline{D} = r_f$$
$$\oint_S \underline{D} \cdot d\underline{S} = \int_V r_f \, dV$$

$\underline{D}$ is not affected by polarisation charges.
$\underline{P}$ exists only within the dielectric.

The total charge density is the sum of the free charge density with the polarisation charge density:

$$r = r_p + r_f$$

Boundary Conditions:

D_{perp}	Continuous across boundaries
$E_{//}$	Continuous across boundaries

$$D_1 \cos q_1 = D_2 \cos q_2$$
$$E_1 \sin q_1 = E_2 \sin q_2$$

The energy density can be given by:

$$U = \int_V \frac{1}{2} \underline{D} \cdot \underline{E} dt$$

Also, Poisson's equation becomes:

$$\nabla^2 f = -\frac{r_f + r_p}{e_0}$$